BAB I

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1.1. DESKRIPSI TUGAS

Selesaikan tugas berikut :

1. Diketahui fungsi f dan g dengan $f(x) = 2x^2 + 3x - 5$ dan $g(x) = 3x - 2$. Tentukan nilai a agar $(gof)(a) = -11$
2. Tentukan turunan pertama dari fungsi $f(x) = \sqrt{\frac{2x}{\sqrt{3x^2+1}}}$
3. Carilah persamaan garis singgung dan garis normal $xy - y + 1 = 0$ disuatu titik dimana koefisien garis singgungnya = 1
4. Hitung :
   a. $\int_{(2-3x^2)^5}^{x} dx$
   b. $\int \frac{(x+3)dx}{(x^2+6x)^{\frac{1}{5}}}$
5. Hitung :
   a. $\int x^2 \cos x \, dx$
   b. $\int_0^{\frac{\pi}{4}} cos^3 2x . \sin 2x \, dx$

1.2. PENYELESAIAN

1.2.1. Tujuan

Tujuan dari tugas ini adalah untuk memahami tentang materi kalkulus terutamana fungsi f dan g, turunan, dan persamaan garis singgung.

1.2.2. Pembahasan

1. $f(x) = 2x^2 + 3x - 5$

   $g(x) = 3x - 2$

(gof) (x) = g (f(x))

= 3 ($2x^2 + 3x – 5$)

= $6x^2 + 9x – 15 -2$

= $6x^2 + 9x – 17$

(g0f) (a) = $6a^2 + 9a – 17$

-11 = 6a2 + 9a -17

0 = 6a2 + 9a -6 (*dibagi 3)*

0 = 2a2 + 3a – 2

0 = (2a – 1) (a + 2)

Maka 2a -1 = 0 atau a + 2 = 0

2a = 1 **a = -2**

a = $\frac{1}{2}$

2. f (x) = $\sqrt{\frac{2x}{\sqrt{3x^2+1}}} = \left(\frac{2x}{(3x^2+1)^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

= $\frac{(2x)^{\frac{1}{2}}}{(3x^2+1)^{\frac{1}{2}\cdot\frac{1}{2}}}$ = $\frac{(2x)^{\frac{1}{2}}}{(3x^2+1)^{\frac{1}{4}}}$

Missal U = $(2x)^{\frac{1}{2}}$

U’ = 2 . $\frac{1}{2}$ = 1

V = $(3x^2+1)^{\frac{1}{4}}$

V’ = $\frac{1}{4}.(3x^2+1)^{\frac{1}{2}}.(6x)$

= $\frac{3}{2}\,x(3x^2+1)^{\frac{1}{2}}$

Maka Turunan f(x)

f(x) = $\frac{U}{V}$

f’(x) = $\frac{U'.V-V'.U}{V^2}$

= $\frac{1.(3x^2+1)^{\frac{1}{4}}-\frac{3}{2}x(3x^2+1)^{\frac{1}{2}}}{((3x^2+1)^{\frac{1}{4}})^2}$

$$= \frac{(3x^2+1)^{\frac{1}{4}-\frac{3}{2}} x(3x^2+1)^{\frac{1}{2}}}{(3x^2+1)^{\frac{1}{2}}}$$

$$= \frac{(3x^2+1)^{\frac{4}{4}}}{(3x^2+1)^{\frac{4}{4}\frac{1}{4}}} - \frac{\frac{3}{2}x(3x^2+1)^{\frac{4}{4}}}{(3x^2+1)^{\frac{4}{4}}}$$

$$= \frac{1}{(3x^2+1)^{\frac{1}{2}}} - \frac{3}{2}x$$

$$= \frac{1}{\sqrt{3x^2+1}} - \frac{3}{3}x$$

$$= \frac{2}{2\sqrt{3x^2+1}} - \frac{3x\sqrt{3x^2+1}}{2\sqrt{3x^2+1}}$$

$$= \frac{2-3x\sqrt{3x^2+1}}{2\sqrt{3x^2+1}}$$

3. $xy - y + 1 = 0$

$y(x-1) = -1$

$y = -\frac{1}{x-1} = \frac{1}{1-x}$

$= (1-x)^{-1}$

$y' = (-(1-x)^{-2}.-1$

$= \frac{1}{(1-x)^2}$

Koefisien garis singgung

$m = 1 = \frac{1}{(1-x)^2}$

$(1-x)^2 = 1$

$(1-x) = 1$

$x = 0$

$0.y - y + 1 = 0$

$y = 1\ titik(0,1)$

PG Singgung

$m = 1\ titik\ (0,1)$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y = x + 1$

PG Normal

$m = -1$ *titik* $(0,1)$

$y - 1 = -1(x - 0)$

$y = -x + 1$

4. a. $\int_{(2-3x^2)^3}^{x} dx$ misal $u = 2 - 3x^2$

$\frac{du}{dx} = -6x$

$dx = \frac{du}{-6x}$

$\int \frac{x}{u^3} \cdot \frac{du}{-6x}$

$\int \frac{1}{-6u^3} du = -\frac{1}{6} \int u^{-3} du$

$= -\frac{1}{6} \cdot \frac{1}{-3+1} \cdot u^{-3+1} + c$

$= -\frac{1}{6} \cdot \frac{1}{-2} \cdot u^{-2} + c$

$= \frac{1}{12} u^{-2} + c$

$= \frac{1}{12\,u^2} + c$

$= \frac{1}{12(2-3x^2)^2} + c$

b. $\int \frac{(x+3)dx}{(x^2+6x)^{\frac{1}{3}}}$ misal $u = x^2$+6x

$\frac{du}{dx} = 2x + 6$

$$dx = \frac{du}{2x+6}$$

$$\int \frac{(x+3)dx}{(x^2+6x)^{\frac{1}{3}}}$$

$$= \int \frac{(x+3)}{u^{\frac{1}{3}}} . \frac{du}{(2x+6)}$$

$$= \frac{1}{2} \int \frac{1}{u^{\frac{1}{3}}} du$$

$$= \frac{1}{2} . \int u^{\frac{1}{3}} du$$

$$= \frac{1}{2} . \int u^{-\frac{1}{3}} du$$

$$= \frac{1}{2} . \frac{1}{-\frac{1}{3}+\frac{1}{2}} . u^{-\frac{1}{3}+\frac{1}{2}} + 6$$

$$= \frac{1}{2} . \frac{1}{\frac{1}{6}} . u^{\frac{1}{6}} + c$$

$$= \frac{1}{2} . \frac{6}{1} . u^{\frac{1}{6}} + c$$

$$= 3(x^2+6x)^{\frac{1}{6}} + c$$

5. a. $\int x^2 \cos x \, dx$

$misal\ u = x^2$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$v = \sin x$$

$$\frac{dv}{dx} = \cos x$$

$$dv = \cos x . dx$$

Maka

$$\int u . dv$$

$$= u . v - \int v . dv$$

$$= x^2 . \sin x - \int \sin x . 2x \; dx$$

$$= x^2 . \sin x - 2 \int x . \sin x . dx$$

$$= x^2 . \sin x + A$$

$$misal \; u = x, du = dx, v = -\cos x, dv = \sin x \; dx$$

Maka

$$2 \int u . dv$$

$$A = -2(u . v - \int v . dv)$$

$$A = -2(x - \cos x - \int -\cos x . dx$$

$$A = -2(-x \cos x + \sin x + c)$$

$$A = 2x \cos x - 2 \sin x + c$$

Hasilnya

$$\int x^2.\cos x\ dx = x^2.\sin x + A$$

$$= x^2.\sin x + 2x\cos x - 2\sin x + c$$

$$= x^2.\sin x - 2\sin x + 2x\cos x + c$$

$$= (x^2 - 2)\sin x + 2x\cos x + c$$

b. $\int_0^{\frac{\pi}{4}} cos^3\ 2x.\sin 2x\ dx$

$misal\ u = \cos 2x$

$$\frac{du}{dx} = -2\sin 2x$$

$$dx = \frac{du}{-2.\sin 2x}$$

$$\int_0^{\frac{\pi}{4}} (4)^3.\sin 2x\ \frac{du}{-2.\sin 2x}$$

$$= -\frac{1}{2}\int_0^{\frac{\pi}{4}} u^3\,du$$

$$= -\frac{1}{2}.\frac{1}{4}u^3\int_0^{u5}$$

$$= -\frac{1}{8}(\cos 2x)^4 + c$$

$$= -\frac{1}{8}(\cos 2\ u5\ ) - \frac{1}{8}(\cos 2.0)$$

$$= -\frac{1}{8}.0 + \frac{1}{8}.1 = 0 + \frac{1}{8} = \frac{1}{8}$$